PETE 693—EXPERIMENTAL METHODS IN PETROLEUM ENGINEERING

Midterm, Thursday 14th November, 2013

Duration: 5:30—7:30pm

Instructions

- **Closed book, closed notes.**
- **Write your answers in this booklet. You might want to use a pencil just in case of erasures.**
- **You need writing material and a simple calculator.**
- **The use of phones and all other types of electronic gadgets is not allowed.**
- **If in doubt about the use of any device, ask.**

Relax, take a deep breath, read through all the questions once and start when you are told to!

1. **(Generalized dimensional analysis-10 points)** Let us suppose the following factors affect the development of fracture conductivity in the laboratory.

$$\left[\sigma, \text{closure stress}\right]$$
$$\left[C_{proppant}, \text{proppant concentration}\right]$$
$$\left[q_{N2}, \text{gas flow rate}\right]$$
$$\left[\mu, \text{viscosity}\right]$$
$$\left[w, width\right]$$
$$\left[F_C, \text{fracture conductivity}\right]$$

   a) Write out the dimensional formulas for the above stated factors using M,L and T as the fundamental units.

b) We follow the generalized dimensional analysis procedure discussed in class and the following matrix is the row-reduced echelon form of the system of equations that define the problem.

$$\begin{pmatrix} a & b & c & d & e & f \\ 1 & 0 & 0 & 0.625 & 0.125 & 0.375 \\ 0 & 1 & 0 & 0.375 & -0.125 & -0.375 \\ 0 & 0 & 1 & -0.25 & -0.25 & -0.75 \end{pmatrix}$$

Note that a, b, c, d, e and f are the exponents of the dimensional formulae for closure stress, proppant concentration, gas flow rate, viscosity, width and fracture conductivity respectively.

i. What is the rank of this matrix?

ii. How many dimensionless variables do you expect to result from this problem?

iii. Write the dimensionless variables and how would you determine the relationship between them.

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2. **(Use of scaling groups-10 points)** In Shook et al.'s paper that we discussed in class, one of the scaling groups they derived is stated below:

$$R_L = \frac{L}{H}\sqrt{\frac{k_z}{k_x}}$$

L is the reservoir length and h is the reservoir thickness. k_z and k_x are the directional permeabilities. You want to make a model of this system in the laboratory. What is your best guess for the length-to-thickness ratio in your laboratory model if $\left(k_z / k_x \right)_{prototype} = 4$ and $\left(k_z / k_x \right)_{model} = 1$?

3. (Probability distributions-10 points)

a) Let the continuous random variable X denote porosity measurements in a reservoir in percentage. Assume that the range of X (porosity) is [0, 40%], and assume that the probability density function of X is $f(x)=0.025$ for $0 \le X \le 40$.

i. What is the probability that a porosity measurement is less than 20%?

ii. What is the probability that a porosity measurement is between 5% and 20%

iii. Plot the probability distribution function.

iv. Plot the cumulative distribution function.

b) The logarithm of a permeability dataset is normally distributed with mean =6.2md and variance=2md^2. We take another measurement and find that the logarithm of that measurement is 11.5 md. Do you think this new measurement belongs to the distribution with mean of 6.2md and variance of 2md^2?

4. (Regression-10 points)

a. We know from class that $y = X\beta$. Based on what you have learnt in class, what is the expression for β keeping in mind that matrix X is not invertible.

b. It has been hypothesized that fracture conductivity is related to closure stress in the following manner where c and f_0 are constants;

$$F_c = \left(\frac{1}{c}\right)\sigma^{1.5} + (f_0)^2 \qquad (1)$$

We also know that if we have data (x_k, y_k) and we think both x and y are linearly related, we can write the following, where m is the slope and b is the intercept;

$$m = \frac{\sum_{k=1}^{N} x_k y_k - \frac{1}{N}\sum_{k=1}^{N} x_k \sum_{k=1}^{N} y_k}{\sum_{k=1}^{N} x_k^2 - \frac{1}{N}\left(\sum_{k=1}^{N} x_k\right)^2} \text{ and}$$

$$b = \frac{1}{N}\left[\sum_{k=1}^{N} y_k - m\sum_{k=1}^{N} x_k\right]$$

Write down expressions for c and f_0 in terms of m and b.

b) Describe how would you compute q_i and D_i from the following equation given that q(t) is production rate at time t and t is time. What is(are) the disadvantage(s) with your approach?

Hint: do not say non-linear regression.

$$q(t) = q_i \exp(-D_i t)$$

c) In linear regression, how do you check if a model is adequate?

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5. **(Factorial and fractional factorial designs-20 points)** In drilling for hydrocarbons, the drilling rate is considered to be affected by 6 variables. It was decided to perform a series of experiments to determine which variables affect the drilling rate. The variables and their levels are shown in the table below.

Variable	Levels		Coded levels	
	Low	**High**	**Low**	**High**
1, Hydrostatic head, psi	6000	9000	-1	+1
2, Type of well	A	B	-1	+1
3, Rotary speed, rpm	100	160	-1	+1
4, Differential pressure, psi	100	500	-1	+1
5, Bit load, lbs/in	2800	4600	-1	+1
6, Overburden pressure, psi	10000	14000	-1	+1

a) Write out a 2^{6-2} fractional factorial design for this experiment (design generators: 5=12, 6=134) Is this design balanced? Give your reason(s).

b) Write out the X -matrix for this design. If the runs were replicated thrice(m=3), what will be the size of matrix X , y and β ? Include **ONLY THE MAIN** effects.

After much thought, we decided to go with a 2^{6-3} factorial design (design generators: 4=-12, 5=-13, 6=123). Accordingly, the following experiments were conducted and we have the table shown below:

Run #	1	2	3	4=-12	5=-13	6=123	Drilling rate, ft/min
1	+1	+1	+1				58
2	-1	+1	+1				64
3	+1	-1	+1				60
4	-1	-1	+1				120
5	+1	+1	-1				102
6	-1	+1	-1				62
7	+1	-1	-1				60
8	-1	-1	-1				66

c) Complete the above table.

d) What is the effect of overburden pressure on the drilling rate?

e) Compute the interaction between rotary speed and differential pressure. Also, make an interaction plot to represent this interaction.

f) Using the **MAIN EFFECTS ONLY**, write a regression model relating drilling rate to the independent parameters without using regression analysis.

g) In a design, we want to be able to estimate at least the main effect and the 2-factor interaction effects uniquely. For an experiment with 6 factors, how many experimental runs do we need to make to be able to estimate the main effects, the 2-factor interations, the intercept and the error uniquely?

Miscellaneous questions (5 points)

The __________________distribution with n degrees of freedom, is the distribution of the random variable

$Z = X_1^2 + X_2^2 + ... + X_n^2$, *where* $X_1,.....,X_n$ *are independently and identically distributed* each with the standard normal distribution N(0,1).

If the p-value is small, we ______________ the __________ hypothesis.

Type I error is _________ the _____________hypothesis when it is _________.

If a model is not adequate, one way to fix it would be to _____________________.

What is the effect heredity principle?

A teacher is never a giver of truth; he/she is a guide, a pointer to the truth that each student must find for him/herself (Bruce Lee). GOOD LUCK!